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w/ Hart & Pillay

Exponential maps for abelian fRM groups and some provably categorical complex analytic structures

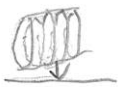
- G comm^ve connected complex Lie group
  $\exp : T_0 G \twoheadrightarrow G$
  For $x \in T_0 G$
  $\alpha \mapsto \exp(\alpha x)$ holomorphic homomorphism
  $\mathbb{C}^+ \to G$
  in particular, homo^ism $\mathbb{Q} \to G$
- G comm^ve connected divisible fRM group $\langle G; +, \dots \rangle$
  $\hat{G} := \{\eta : \mathbb{Q} \xrightarrow{\text{homo}^n} G\}$
  $\widehat{\exp} : \hat{G} \twoheadrightarrow G$
  $; \eta \to \eta(1)$
  $\hat{T}_G := Th(\hat{G} \overset{\widehat{\exp}}{\twoheadrightarrow} G)$
  where $\hat{G}$ is $\mathbb{Q}$-VS w/ predicate for $\hat{H} \leq \hat{G}^n$ $\hat{H} = \{\eta : \mathbb{Q} \to H\}$ $\left((q\cdot\eta)(x) = \eta(q\cdot x)\right)$
  for $H \leq G^n$ connected defble
  (assume all such $H/\emptyset$)
  ("rigid")
- QE
- Analytic models:

  (I) G complex semiab^n vary
  $(LG \overset{\exp}{\twoheadrightarrow} G) \models \hat{T}_G$ $\left(\begin{array}{l}\eta : \mathbb{Q} \to G) \in LG \leq \hat{G} \\ \text{iff } \lim_{q \to 0} \eta(q) = 0 \in G\end{array}\right)$

  (II) G complex torus i.e. compact complex Lie group
  in CCM language (more generally: G a comm^ve rigid mero^c grp)
  $\exp \models T_G$

  (III) $A / \mathbb{C}(t)$ ab^n vary (e.g. $y^2 = x(x-1)(x-t)$)
  S disc ~~…~~ A_s ab^n vary $\forall s \in S$
  $L := \{\text{mero}^c \text{ f}^{ns} \text{ on } S\} \geq \mathbb{C}(t)$
  $LA(L) \overset{\exp}{\longrightarrow} A(L)$ "relative exp^t"
  $LA^\#(L) := \langle \ker \exp \rangle_{\mathbb{C}}$ $A^\#(L) := \exp(LA^\#(L))$

$A^\#$ admits diff^c alg^c description:



$U \hookrightarrow \tilde{A} \twoheadrightarrow A$ univ^l vect^r ext^n
$\underset{\exp}{\uparrow}$ $\uparrow \exp$
$L\tilde{A} \twoheadrightarrow LA$
unique D-structure on $\tilde{A}$, $L\tilde{A}$ (algebraic lift of the vector field $\frac{d}{dt}$ on S)
$\tilde{A}^\partial, L\tilde{A}^\partial :=$ horizontal points

$$\begin{array}{ccc} L\tilde{A}^\partial(L) & \twoheadrightarrow & LA^\#(L) \\ \exp \downarrow & & \downarrow \exp \\ \tilde{A}^\partial(L) & \twoheadrightarrow & A^\#(L) \end{array}$$

$\boxed{L\tilde{A}^\partial = \langle \ker \exp_{\tilde{A}} \rangle_{\mathbb{C}}}$

$\boxed{A^\# := \theta(\tilde{A}^\partial)}$

Fact: $A^\#(L) = A^\#(L^{\text{diff}})$ "Manin kernel" fRM group
and $(LA^\#(L) \overset{\exp}{\twoheadrightarrow} A^\#(L)) \models \hat{T}_{A^\#}$
with structure induced from DCF

~~…~~

- Classification Th^m:
  G comm^ve connected divisible rigid fRM group
  Models $(\exp : \hat{M} \to M) \models \hat{T}_G$
  are determined up to IM by IM types of
  - $M \models Th(G)$
  - $\hat{M}_0 := \exp^{-1}(M_0)$ where $M_0 \preceq M$ prime

  Pf sketch: $B :=$ bases for sm sets in M
  show $\hat{M}$ contractible $/\hat{M}_0 B$.
  For B finite: $\Leftarrow$ [BGH]
  Generally: "NOTOP arguments" □

- $\hat{M}_0$ and Kummer th^y:
  $G = A \times G_m^n$ A ab^n vary / k_0 n.f
  Faltings $\Rightarrow$ $\hat{M}_0 = \hat{G}(\bar{\mathbb{Q}})$ atomic / ker exp
  so $LG(\mathbb{C}) \overset{\exp}{\twoheadrightarrow} G(\mathbb{C})$ characterised by:
  - $LG(\mathbb{C})$ divisible t.f. End(G)-module
  - exp surj^ve End(G)-HM
  - $\langle \mathbb{C}; +, \cdot \rangle$ ACF$/k_0$ card^y $2^{\aleph_0}$
  - $\mathbb{Q} \otimes (\ker \exp) \overset{\exp}{\twoheadrightarrow} \text{Tor}\, G$ is what it is.

Same goes for Manin kernels of non-isoconstant A, since prime model is Tor(A)!