Accompanying notes for the slides "Generalizing the Hardy-Littlewood method for primes".

These brief notes are intended to accompany the slides for my lecture at the ICM in Madrid, August 2006. They are probably a good way of gaining some understanding of what the paper [4] is about. The slides contain certain inaccuracies which I introduced deliberately for the sake of expositional clarity. Anyone wishing to use these slides for independent study would not have the benefit of having me explain verbally where those inaccuracies lie. These notes draw attention to them.

The reader who is seriously interested in the subject should, of course, look at the paper [4]. The first chapter of that paper is expository and should be reasonably easy to follow. My ICM article [1] gives a bit more detail than these slides, though it was written about a year before the paper [4] and has a slightly different perspective. Most notably, it talks about linear equations rather than linear forms.

Slide 15. The actual decomposition $\Lambda = \Lambda^\sharp + \Lambda^\flat$ involves chopping the range of divisors d into two parts using a smooth cutoff. See [4, §12] for details.

Slide 16. (Also subsequent slides.) At this point we see the introduction of $\mathbb{Z}/N\mathbb{Z}$, where before we had been talking about $\{1, \dots, N\}$. When one discusses the Gowers norms it is advantageous to work in a *group*, as various averaging devices then become easier. There are various technical devices which allow one to transfer problems in $\{1, \dots, N\}$ to $\mathbb{Z}/N\mathbb{Z}$, but in anything other than a very technical talk it is best to suppress them. This technicality first enters [4] in §7. For examples of the transition between $\{1, \dots, N\}$ and $\mathbb{Z}/N\mathbb{Z}$, see [4, Appendix C].

Slide 18. Note that in our recap of the definition of $T(f_1, \dots, f_t)$ we have quietly dropped the convex body K, and moved to $\mathbb{Z}/N\mathbb{Z}$.

The correct definition of "reasonably general" in the context of the Generalised von Neumann Theorem is that f needs to be bounded by a *pseudorandom measure*. See [4, §6,7].

Slides 20–24. For detailed statements and proofs of these results, see [2].

Slide 25. It is not at all clear what is meant by the "Lipschitz constant of F". For an explanation, see [4, §8].

Slide 26. By "somewhat modified version of Λ" we mean that Λ must first be subjected to the so-called W-trick, which removes biases in residue classes to small moduli. See [4, §5]. In fact this modification must really be made at a fairly early stage of the discussion, when Λ is decomposed as $\Lambda^\sharp + \Lambda^\flat$.

Slide 29. Here "small" means $o(1)$, whilst "really rather small" means $O_A(\log^{-A} N)$ for any $A > 0$. This is all explained in [4].

Slide 30. For the proof of the MN(2) conjecture, see [3].

References

[1] B. J. Green *Generalizing the Hardy-Littlewood method for primes,* Proc. Internat. Cong. Math., Madrid 2006.

[2] B. J. Green and T. C. Tao, *An inverse theorem for the Gowers U^3-norm,* to appear in Proc. Edin. Math. Soc.

[3] B. J. Green and T. C. Tao, *Quadratic uniformity of the Möbius function,* preprint.

[4] B. J. Green and T. C.Tao, *Linear equations in primes,* preprint.