Reconstructing 3D Cube Complexes from Boundary Distances

Jane Tan (Oxford)
with Haslegrave, Scott and Tamitegama

From differential geometry.

From differential geometry.

Def$^{\underline{n}}$ A Riemannian manifold (M, g) is **boundary rigid** if its associated metric $d_g : M \times M \to \mathbb{R}$ is determined up to isometry by its boundary distance function $d_g|_{\partial M \times \partial M}$.

From differential geometry.

Def$^{\underline{n}}$ A Riemannian manifold (M, g) is **boundary rigid** if its associated metric $d_g : M \times M \to \mathbb{R}$ is determined up to isometry by its boundary distance function $d_g|_{\partial M \times \partial M}$.

Conjecture (Michel 1981)

Every simple compact Riemannian manifold is boundary rigid.

From differential geometry

Defn A Riemannian manifold (M, g) is **boundary rigid** if its associated metric $d_g : M \times M \to \mathbb{R}$ is determined up to isometry by its boundary distance function $d_g|_{\partial M \times \partial M}$.

Conjecture (Michel 1981)
Every simple compact Riemannian manifold is boundary rigid.

Known for
- surfaces (Pestov, Uhlmann 2005)
- simple subspaces of $\mathbb{R}^n$ (Gromov 1991)
- symmetric spaces of constant negative curvature (Besson, Courtois, Gallot 1995)

From differential geometry.

Def$^{\underline{n}}$ A Riemannian manifold (M, g) is boundary rigid if its associated metric $d_g : M \times M \to \mathbb{R}$ is determined up to isometry by its boundary distance function $d_g|_{\partial M \times \partial M}$.

Conjecture (Michel 1981)
Every simple compact Riemannian manifold is boundary rigid.

Known for
- surfaces (Pestov, Uhlmann 2005)
- simple subspaces of $\mathbb{R}^n$ (Gromov 1991)
- symmetric spaces of constant negative curvature (Besson, Courtois, Gallot 1995)

Generally wide open in dimensions ≥ 3

A discrete version

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

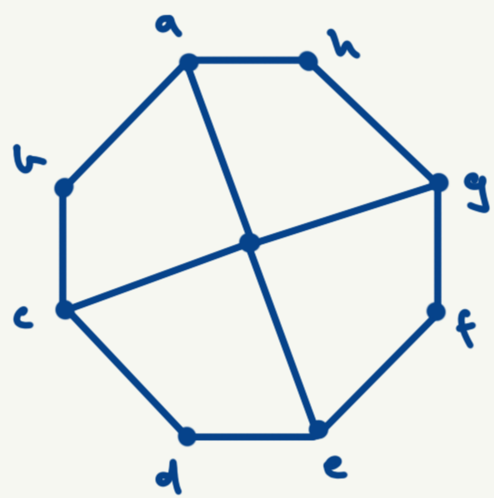
$$
\begin{array}{cc}
 & \begin{matrix} a & b & c & d & e & f & g & h \end{matrix} \\
\begin{matrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{matrix} &
\begin{bmatrix}
0 & 1 & 2 & 3 & 2 & 3 & 2 & 1 \\
1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 \\
2 & 1 & 0 & 1 & 2 & 3 & 2 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\
2 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\
3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\
2 & 3 & 2 & 3 & 2 & 1 & 0 & 1 \\
1 & 2 & 3 & 4 & 3 & 2 & 1 & 0
\end{bmatrix}
\end{array}
$$

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

$$
\begin{array}{c c}
 & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} &
\begin{bmatrix}
0 & 1 & 2 & 3 & 2 & 3 & 2 & 1 \\
1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 \\
2 & 1 & 0 & 1 & 2 & 3 & 2 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\
2 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\
3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\
2 & 3 & 2 & 3 & 2 & 1 & 0 & 1 \\
1 & 2 & 3 & 4 & 3 & 2 & 1 & 0
\end{bmatrix}
\end{array}
\longrightarrow
$$

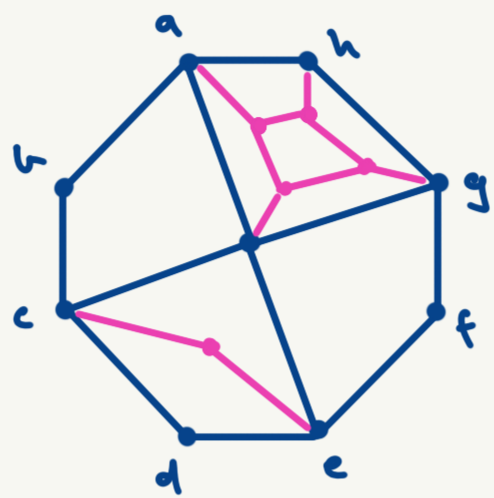


A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

$$
\begin{array}{c}
\begin{array}{cc} & \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} &
\begin{bmatrix}
0 & 1 & 2 & 3 & 2 & 3 & 2 & 1 \\
1 & 0 & 1 & 2 & 3 & 4 & 3 & 2 \\
2 & 1 & 0 & 1 & 2 & 3 & 2 & 3 \\
3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\
2 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\
3 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\
2 & 3 & 2 & 3 & 2 & 1 & 0 & 1 \\
1 & 2 & 3 & 4 & 3 & 2 & 1 & 0
\end{bmatrix}
\end{array}
\end{array}
\longrightarrow
$$



Distances between boundary vertices remain the same

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

Goal: move up a dimension

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

cube complex

Aside

Cube complex essentials

Dimension	0	1	2	3	n
simplex					convex hull of n+1 pts
cube					I^n

Aside

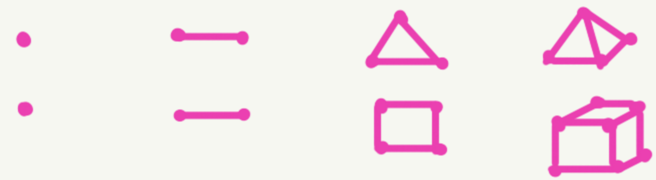
Cube complex essentials

Dimension	0	1	2	3	n
simplex	•				convex hull of $n+1$ pts
cube	•				I^n

<u>Def$^{\underline{n}}$</u> A cube complex is a cell complex obtained by gluing cubes together along faces.

Aside

Cube complex essentials



Dimension	0	1	2	3	n
simplex					convex hull of $n+1$ pts
cube					I^n

Def$^{\underline{n}}$ A **cube complex** is a cell complex obtained by gluing cubes together along faces.

Its **dimension** is the dimension of the top cube.

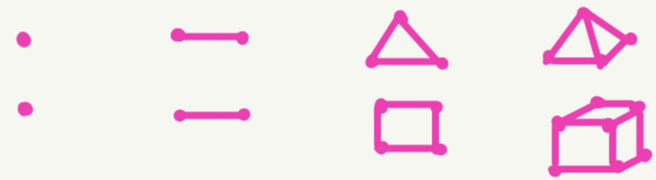
It is **pure** if every cube is in a top-dimensional cube.

The **k-skeleton** is the union of all cubes with dim $\leq k$.

Aside

Cube complex essentials

Dimension	0	1	2	3	n
simplex					convex hull of $n+1$ pts
cube					I^n



Def$^{\underline{n}}$ A cube complex is a cell complex obtained by gluing cubes together along faces.

Its dimension is the dimension of the top cube.

It is pure if every cube is in a top-dimensional cube.

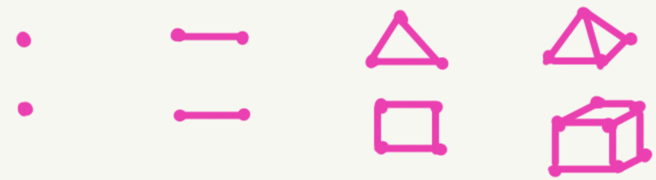
The k-skeleton is the union of all cubes with dim $\leq k$.

Note that
- the intersection of any two cubes is another cube in the complex
- the boundary of a k-dimensional cube complex is a $(k-1)$-dimensional cube complex

Aside

Cube complex essentials

Dimension	0	1	2	3	n
simplex					convex hull of n+1 pts
cube					I^n



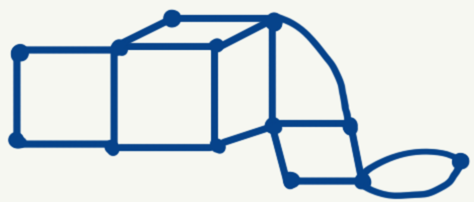
Def$^{\underline{n}}$ A cube complex is a cell complex obtained by gluing cubes together along faces.

Its dimension is the dimension of the top cube.

It is pure if every cube is in a top-dimensional cube.

The k-skeleton is the union of all cubes with dim $\leq k$.

Note that
- the intersection of any two cubes is another cube in the complex
- the boundary of a k-dimensional cube complex is a $(k-1)$-dimensional cube complex



Aside

Cube complex essentials

Dimension	0	1	2	3	n
simplex					convex hull of n+1 pts
cube					I^n

(In 3D: vertex edge face cube)

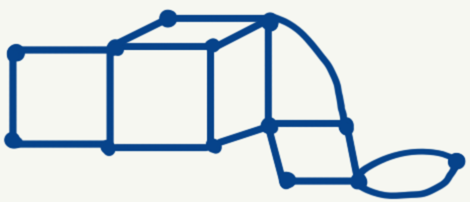
Def^n A **cube complex** is a cell complex obtained by gluing cubes together along faces.

Its **dimension** is the dimension of the top cube.

It is **pure** if every cube is in a top-dimensional cube.

The **k-skeleton** is the union of all cubes with dim $\leq k$.

Note that
- the intersection of any two cubes is another cube in the complex
- the boundary of a k-dimensional cube complex is a $(k-1)$-dimensional cube complex



A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

cube complex

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

cube complex

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4.
Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

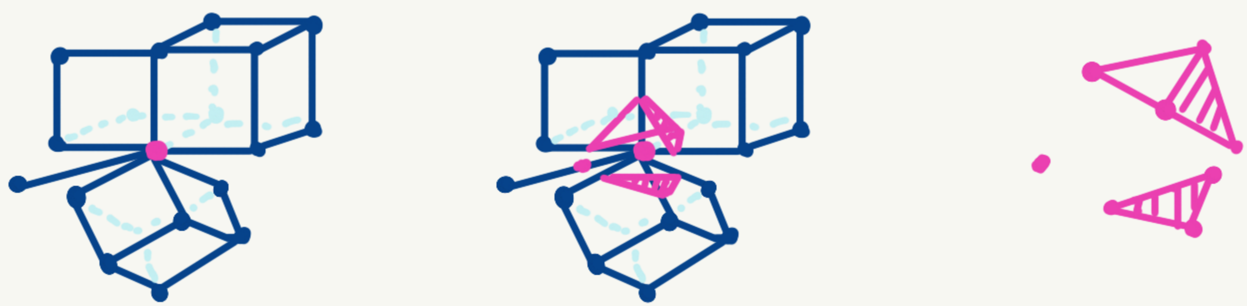
CAT(0) cube complex

Aside

Cube complex essentials

Def$^{\underline{n}}$ The **link** of a vertex in a cube complex is the simplex complex with n-simplices $\leftrightarrow$ corners of $(n+1)$-cubes.

(Alternatively, intersection of $B_\varepsilon(v)$ with complex)

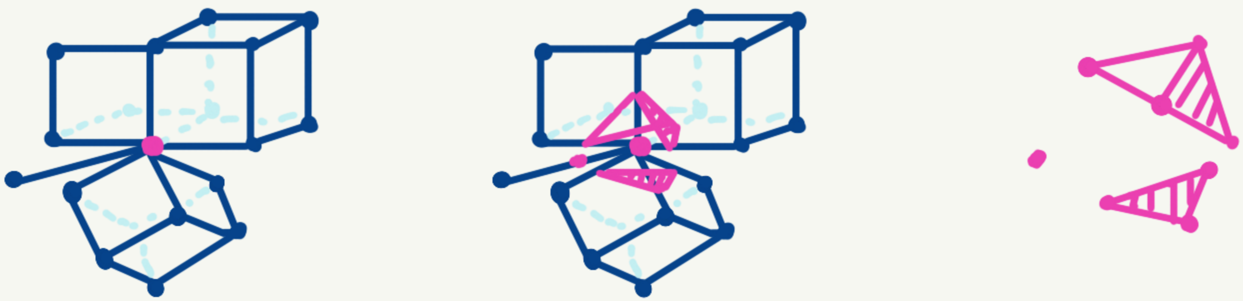


Aside

Cube complex essentials

Def$^{\underline{n}}$ The **link** of a vertex in a cube complex is the simplex complex with n-simplices ↔ corners of (n+1)-cubes.

(Alternatively, intersection of $B_\varepsilon(v)$ with complex)



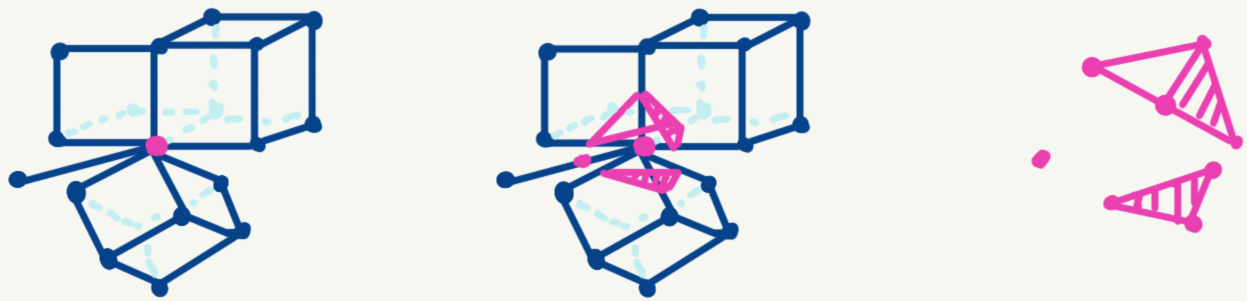
A simplicial complex is **flag** if n+1 vertices span a simplex iff they are pairwise adjacent.

Aside

Cube complex essentials

Def$^{\underline{n}}$ The link of a vertex in a cube complex is the simplex complex with n-simplices $\leftrightarrow$ corners of $(n+1)$-cubes.

(Alternatively, intersection of $B_\varepsilon(v)$ with complex)



A simplicial complex is flag if $n+1$ vertices span a simplex iff they are pairwise adjacent.

A cube complex is CAT(0) if it is simply connected and the link of every vertex is a flag simplicial complex.

"Gromov's link condition"

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4.

Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

CAT(0) cube complex

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

CAT(0) cube complex

Claim: CAT(0) $\Rightarrow$ contractible

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

CAT(0) cube complex

Claim: CAT(0) $\Rightarrow$ contractible

In 2D: CAT(0) $\Rightarrow$ all internal degrees ≥ 4

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

CAT(0) cube complex

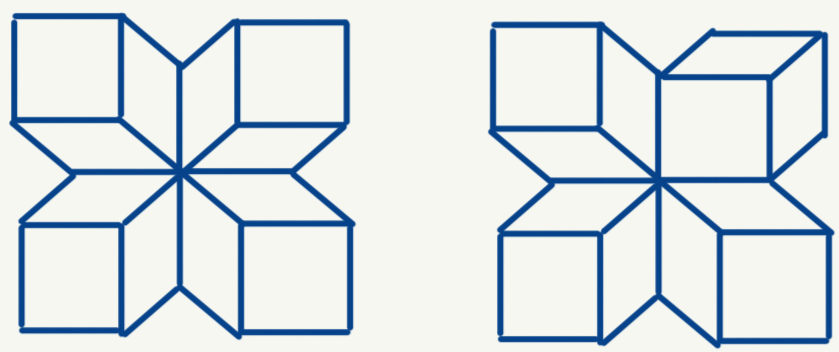
A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

up to combinatorial type

CAT(0) cube complex



A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

CAT(0) cube complex

up to combinatorial type

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

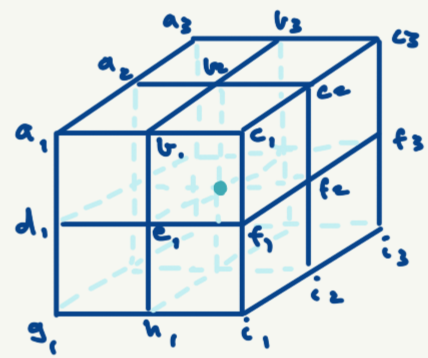
CAT(0) cube complex

up to combinatorial type

between vertices on ∂

matrix of pairwise distances

$$\begin{array}{c|cccccc} & a_1 & a_2 & a_3 & \cdots & & i_3 \\ \hline a_1 & & 1 & 2 & & & 6 \\ a_2 & & & 1 & & & 5 \\ a_3 & & & & & & 4 \\ \vdots & & & & & & \\ i_3 & & & & & & \end{array} \quad \longrightarrow$$



A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

CAT(0) cube complex

up to combinatorial type

matrix of pairwise distances between vertices on ∂

A discrete version

Theorem (Haslegrave 2021)

Suppose that Q is a quadrangulation with simple closed boundary such that all internal vertices have degree ≥ 4. Then Q is reconstructible up to graph isomorphism from the distances between boundary vertices of Q.

Theorem (Haslegrave, Scott, Tamitegama, T. 2023+)

Suppose that X is a finite CAT(0) cube complex which admits an embedding in $\mathbb{R}^3$. Then X is reconstructible up to combinatorial type from the matrix of pairwise distances between vertices on ∂X.

The dream proof.

We would like to proceed by induction on the size of the complex.

The dream proof.

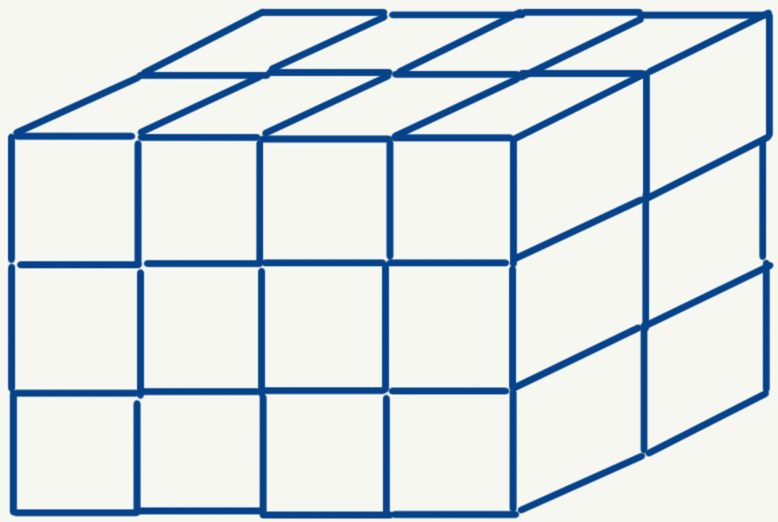
We would like to proceed by induction on the size of the complex.

Idea: find and remove a row of cubes in each step, apply induction hypothesis.

The dream proof.

We would like to proceed by induction on the size of the complex.

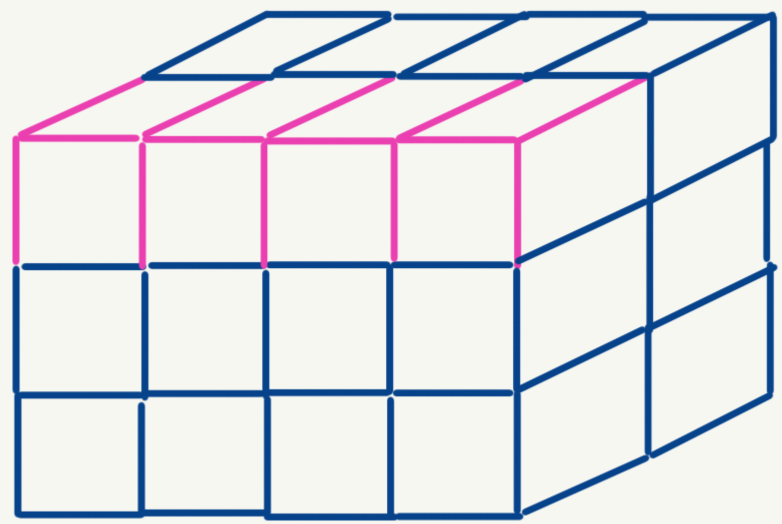
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

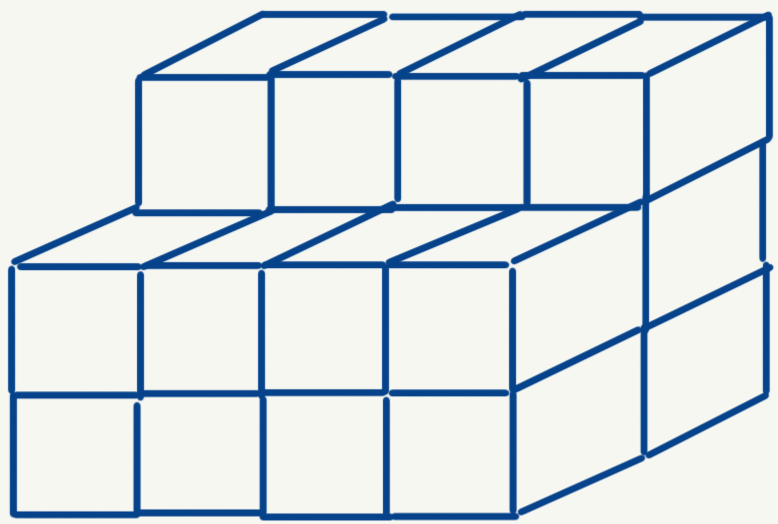
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

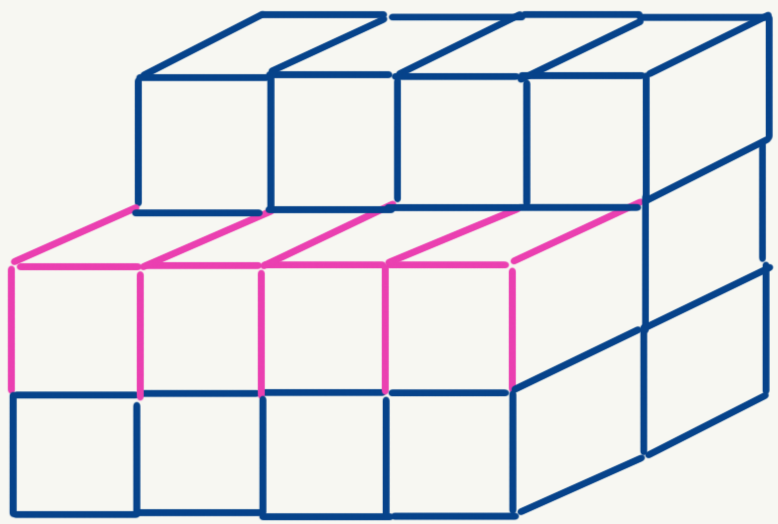
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

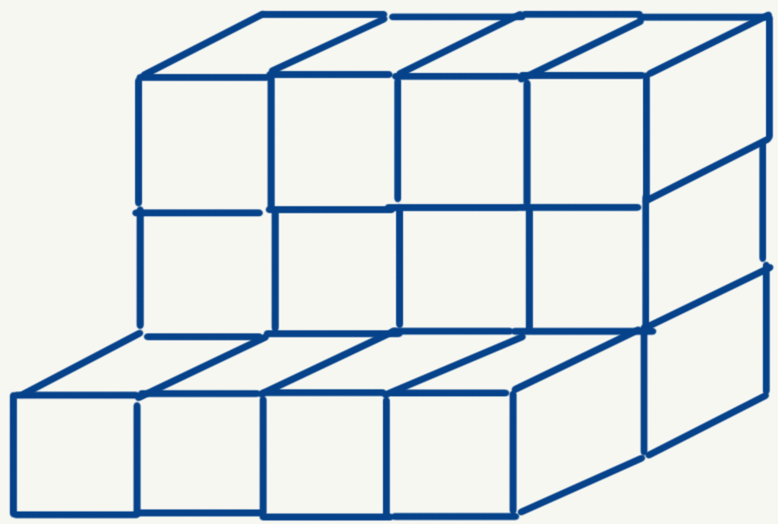
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

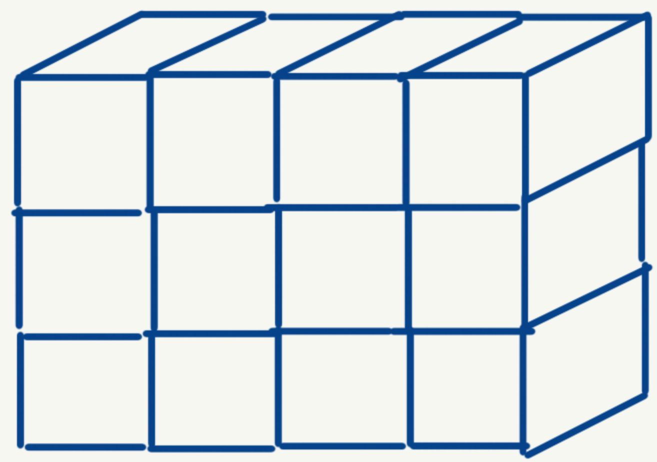
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

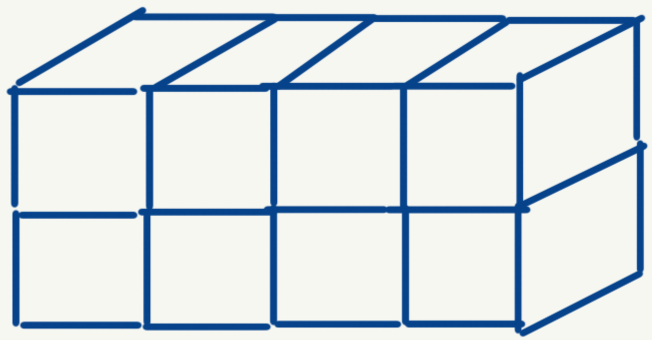
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

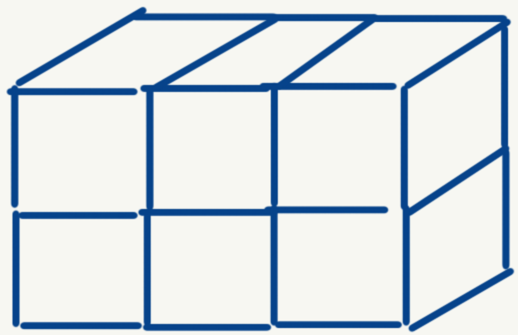
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

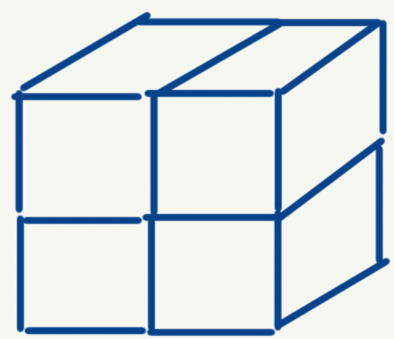
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

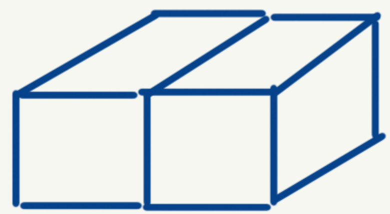
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

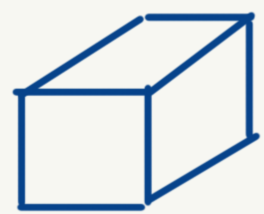
Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex.

Idea: find and remove a row of cubes in each step, apply induction hypothesis.



The dream proof.

We would like to proceed by induction on the size of the complex X.

Idea: find and remove a row of cubes in each step, apply induction hypothesis.

Recognition lemma:

Row of cubes

we can recognise from the matrix of distances which boundary vertices are in a row of cubes.

Existence lemma: X contains a row of cubes.

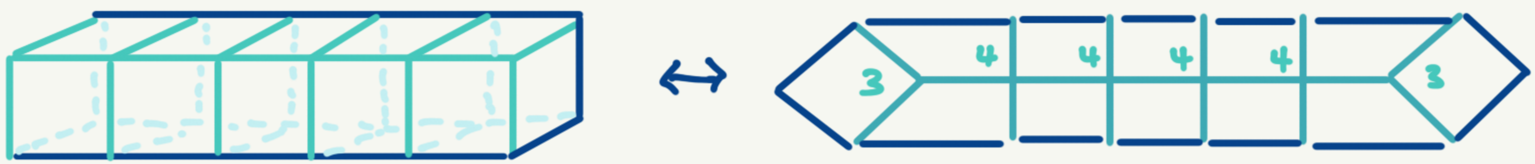
Removal lemma: $X - R$ is CAT(0) and we can recover the matrix of boundary distances for $X - R$.

The dream proof.

We would like to proceed by induction on the size of the complex X.

Idea: find and remove a row of cubes in each step, apply induction hypothesis.

✓ Recognition lemma:



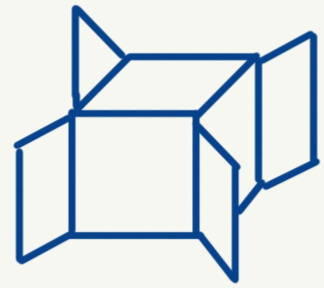
Row of cubes $\leftrightarrow$ Good row configuration in boundary graph

Existence lemma: X contains a row of cubes.

Removal lemma: $X - R$ is CAT(0) and we can recover the matrix of boundary distances for $X - R$.

There are cube complexes where :

- there are no rows of cubes



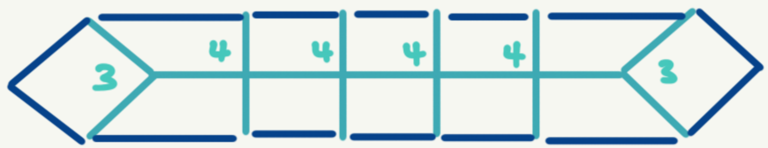
The dream proof.

We would like to proceed by induction on the size of the complex X.

Idea: find and remove a row of cubes in each step, apply induction hypothesis.

✓ Recognition lemma:

$\leftrightarrow$



Existence lemma: If X is homeomorphic to the 3-ball, then it contains a row of cubes.

Removal lemma: $X - R$ is CAT(0) and we can recover the matrix of boundary distances for $X - R$.

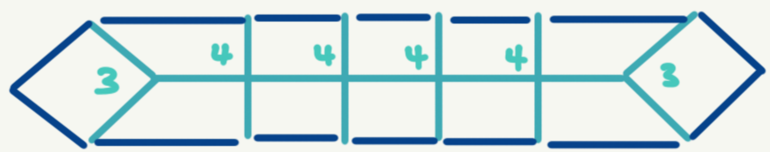
The dream proof.

We would like to proceed by induction on the size of the complex X.

Idea: find and remove a row of cubes in each step, apply induction hypothesis.

✓ Recognition lemma:

$\leftrightarrow$



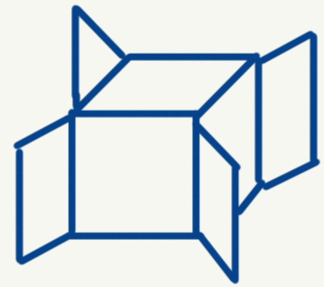
Row of cubes

Good row configuration in boundary graph

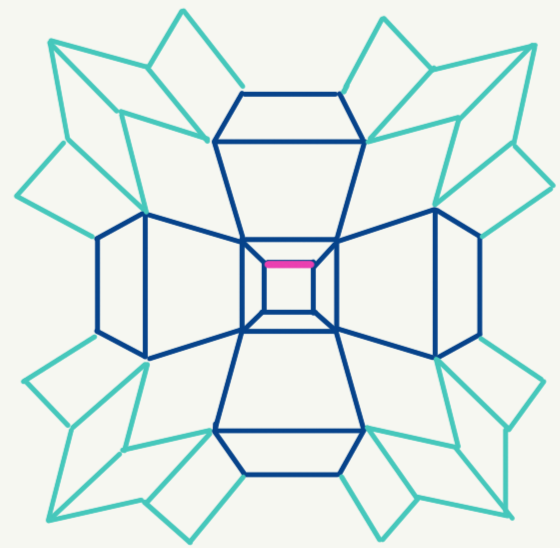
✓ Existence lemma: If X is homeomorphic to the 3-ball, then it contains a row of cubes.

Removal lemma: $X - R$ is CAT(0) and we can recover the matrix of boundary distances for $X - R$.

There are cube complexes where:



- there are no rows of cubes



- $X - R$ is not CAT(0) (depending on how removal is defined)

The dream proof.

We would like to proceed by induction on the size of the complex X.

Idea: find and remove a **row of cubes** in each step, apply induction hypothesis.

✓ **Recognition lemma:**

$\leftrightarrow$

Row of cubes — Good row configuration in boundary graph

✓ **Existence lemma:** If X is homeomorphic to the 3-ball, then it contains a row of cubes.

Removal lemma: Sp R is a row of cubes in X and $X - R$ is simply connected. Then $X - R$ is CAT(0) and we can recover the matrix of boundary distances for $X - R$.

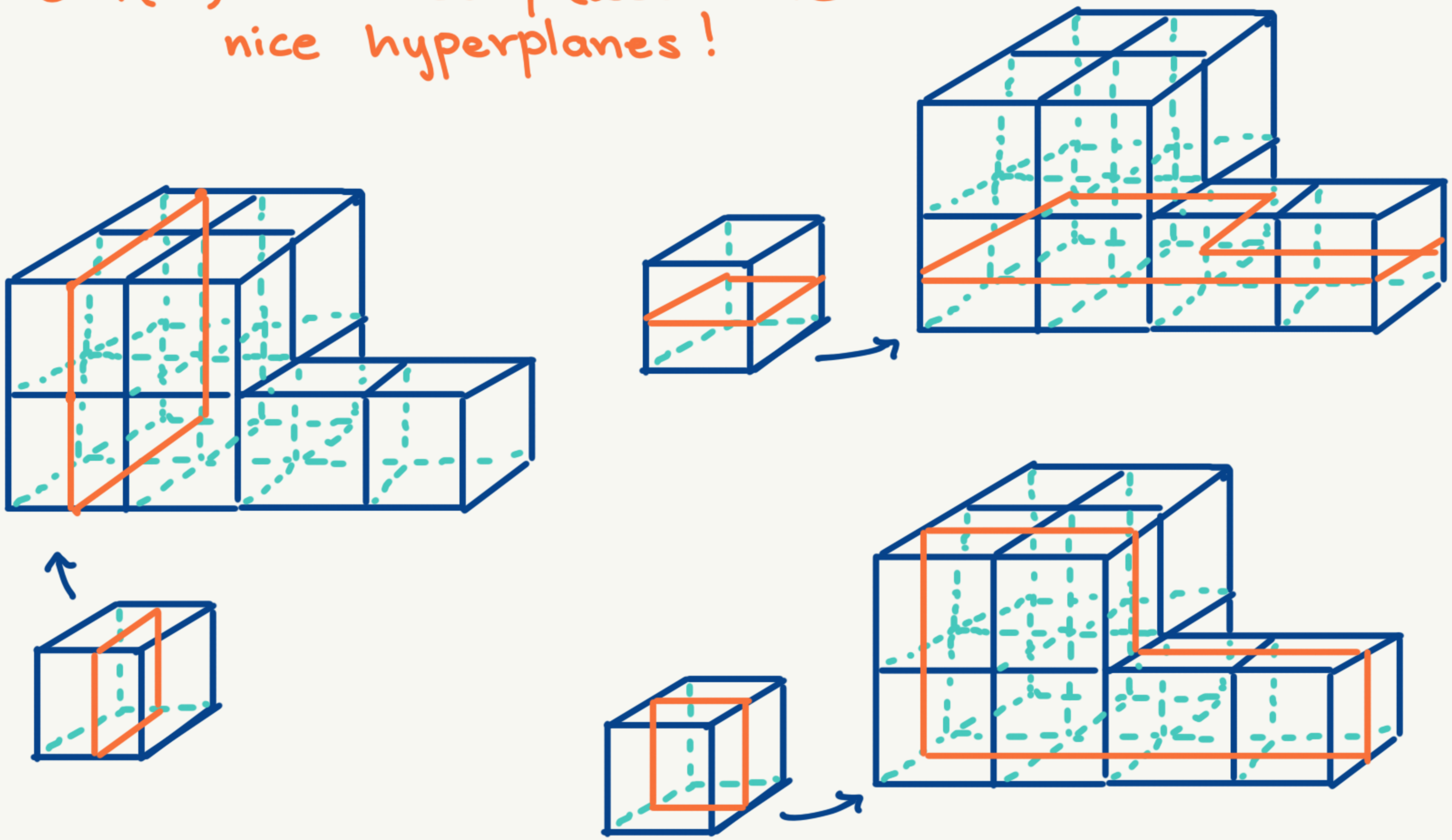
The dream proof.

Claim: We can recover the matrix of boundary distances for $X - R$.

The dream proof.

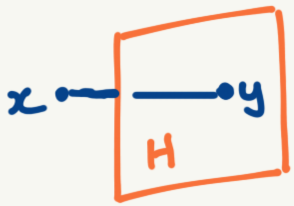
Claim: We can recover the matrix of boundary distances for $X - R$.

CAT(0) cube complexes have nice hyperplanes!



The dream proof.

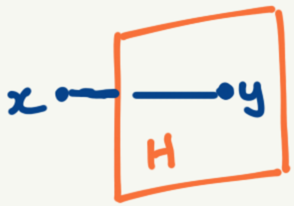
Claim: We can recover the matrix of boundary distances for $X - R$.

Key lemma: sp  in X. Then $\forall\, v \in X$,

$$d_X(v, x) = d_X(v, y) + 1 \iff v \text{ is on the side of } H \text{ containing } y.$$

The dream proof.

Claim: We can recover the matrix of boundary distances for $X - R$.

Key lemma: sp 

in X. Then $\forall v \in X$,

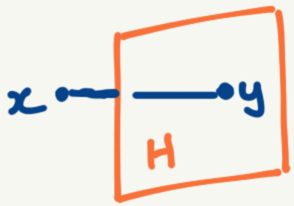
$d_X(v, x) = d_X(v, y) + 1 \iff$ v is on the side of H containing y.

This means:

- If $x, y \in \partial X$, we can determine which side of H any point of ∂X lies.
- If $x \in \partial X$ and we know which side of H v lies, then we can determine $d_X(v, y)$.

The dream proof.

Claim: We can recover the matrix of boundary distances for $X - R$.

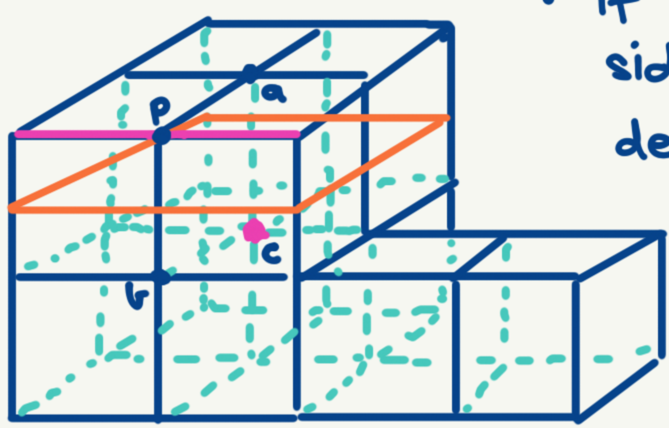
Key lemma: sp 

in X. Then $\forall v \in X$,

$d_X(v,x) = d_X(v,y) + 1 \Leftrightarrow$ v is on the side of H containing y.

This means:

- If $x, y \in \partial X$, we can determine which side of H any point of ∂X lies.
- If $x \in \partial X$ and we know which side of H v lies, then we can determine $d_X(v,y)$.



The dream proof.

We would like to proceed by induction on the size of the complex X.

Idea: find and remove a row of cubes in each step, apply induction hypothesis.

✓ Recognition lemma:

↔

Row of cubes

Good row configuration in boundary graph

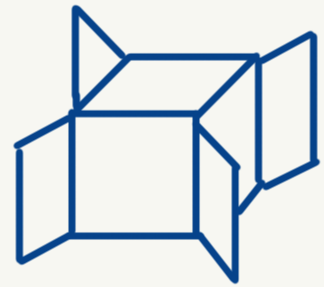
✓ Existence lemma: If X is homeomorphic to the 3-ball, then it contains a row of cubes.

✓ Removal lemma: Sp R is a row of cubes in X and X-R is simply connected. Then X-R is CAT(0) and we can recover the matrix of boundary distances for X-R.

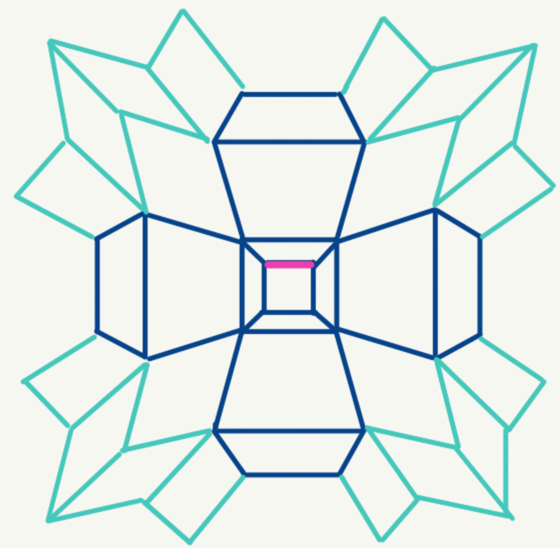
<u>But let's not forget ...</u>

There are cube complexes where:

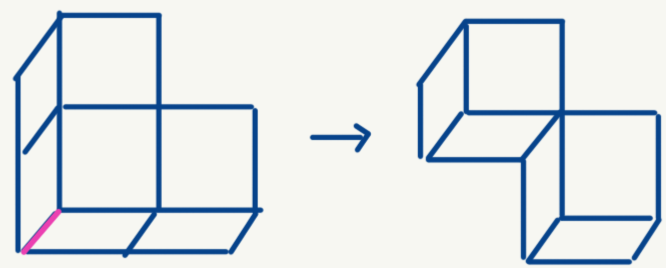
- there are no rows of cubes



- $X-R$ is not CAT(0)
  (depending on how removal is defined)



- $X-R$ is not homeomorphic to a ball



Corrected proof

Recall

> Suppose that X is a finite CAT(0) cube complex which admits an embedding in $\mathbb{R}^3$. Then X is reconstructible up to combinatorial type from the matrix of pairwise distances between vertices on ∂X.

↳ not necessarily pure, not necessarily $\cong B^3$

Corrected proof

Recall

[Suppose that X is a finite CAT(0) cube complex which admits an embedding in $\mathbb{R}^3$. Then X is reconstructible up to combinatorial type from the matrix of pairwise distances between vertices on ∂X.]

We consider 4 substructures and corresponding reductions.

Corrected proof

Recall: [Suppose that X is a finite CAT(0) cube complex which admits an embedding in $\mathbb{R}^3$. Then X is reconstructible up to combinatorial type from the matrix of pairwise distances between vertices on ∂X.]

We consider 4 substructures and corresponding reductions.

1. Cut-vertices

Corrected proof

Recall: [Suppose that X is a finite CAT(0) cube complex which admits an embedding in $\mathbb{R}^3$. Then X is reconstructible up to combinatorial type from the matrix of pairwise distances between vertices on ∂X.]

We consider 4 substructures and corresponding reductions.

① Cut-vertices

② corner of a face

Corrected proof

Recall

[Suppose that X is a finite CAT(0) cube complex which admits an embedding in $\mathbb{R}^3$. Then X is reconstructible up to combinatorial type from the matrix of pairwise distances between vertices on ∂X.]

We consider 4 substructures and corresponding reductions.

① Cut-vertices

② Corner of a face

③ Degree 3 vertex not in a cube

Corrected proof

Recall: Suppose that X is a finite CAT(0) cube complex which admits an embedding in $\mathbb{R}^3$. Then X is reconstructible up to combinatorial type from the matrix of pairwise distances between vertices on ∂X.

We consider 4 substructures and corresponding reductions.

1. Cut-vertices

2. Corner of a face

3. Degree 3 vertex not in a cube

4. Row of cubes

Corrected proof

① Cut-vertices

② corner of a face

③ Degree 3 vertex not in a cube

④ Row of cubes

Structure lemma: Every finite CAT(0) cube complex embeddable in $\mathbb{R}^3$ is either an edge or contains one of these 4 structures.

Corrected proof

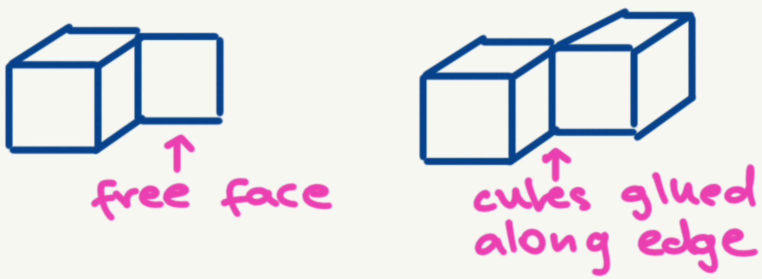
① Cut-vertices

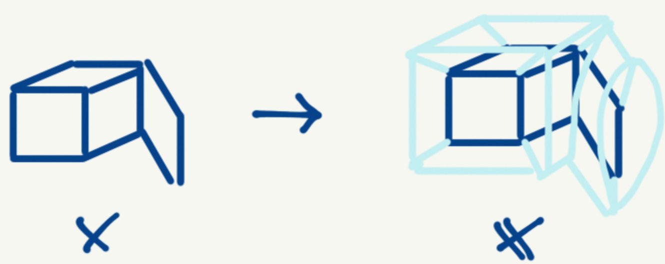
② corner of a face

③ Degree 3 vertex not in a cube

④ Row of cubes

Structure lemma : Every finite CAT(0) cube complex embeddable in $\mathbb{R}^3$ is either an edge or contains one of these 4 structures.





Corrected proof

① Cut-vertices

② corner of a face

③ Degree 3 vertex not in a cube

④ Row of cubes

Structure lemma: Every finite CAT(0) cube complex embeddable in $\mathbb{R}^3$ is either an edge or contains one of these 4 structures.

Recognition lemma: Each substructure can be identified from the matrix of boundary distances.

Corrected proof

① Cut-vertices

② corner of a face

③ Degree 3 vertex not in a cube

④ Row of cubes

Structure lemma: Every finite CAT(0) cube complex embeddable in $\mathbb{R}^3$ is either an edge or contains one of these 4 structures.

Recognition lemma: Each substructure can be identified from the matrix of boundary distances.

Removal lemma: Each reduction operation leaves a CAT(0) cube complex (contractible + flag) for which we can recover the matrix of boundary distances.

Corrected proof (Reduction)

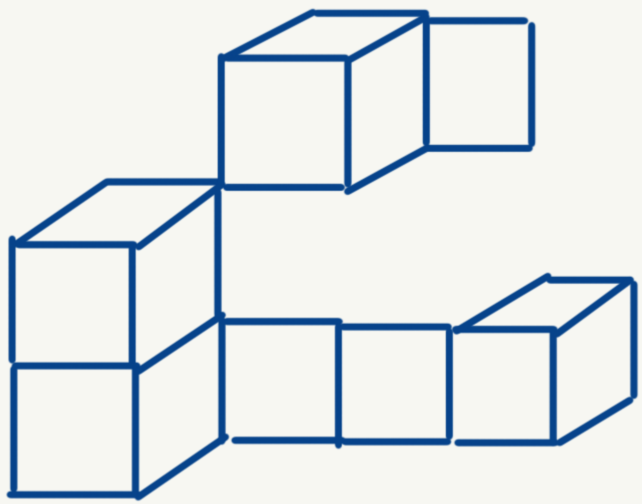
Procedure : While X is not an edge,

Apply ① if possible, then iterate on components.

Else apply ② if possible, then iterate.

Else apply ③ if possible, then iterate.

Else apply ④, then iterate.



Corrected proof (Reduction)

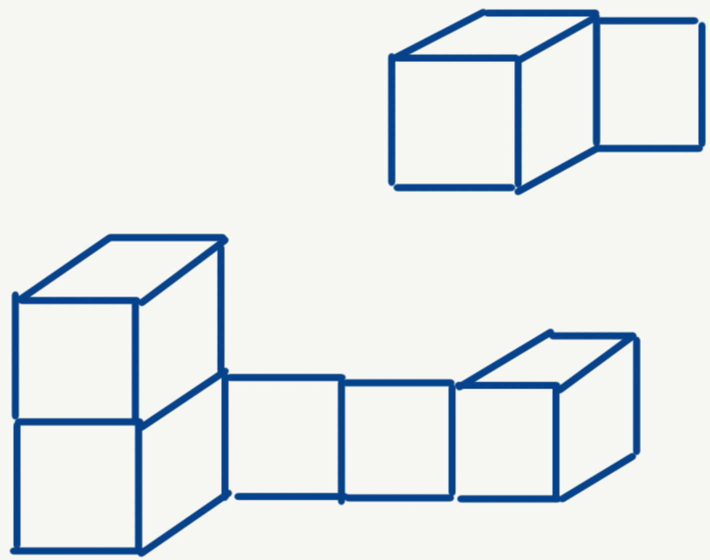
Procedure : While X is not an edge,

Apply ① if possible, then iterate on components.

Else apply ② if possible, then iterate.

Else apply ③ if possible, then iterate.

Else apply ④, then iterate.



Corrected proof (Reduction)

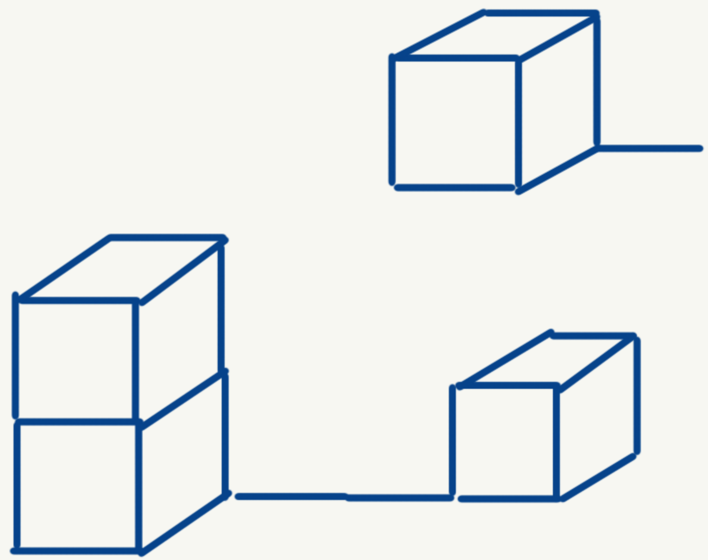
Procedure : While X is not an edge,

Apply ① if possible, then iterate on components.

Else apply ② if possible, then iterate.

Else apply ③ if possible, then iterate.

Else apply ④, then iterate.



Corrected proof (Reduction)

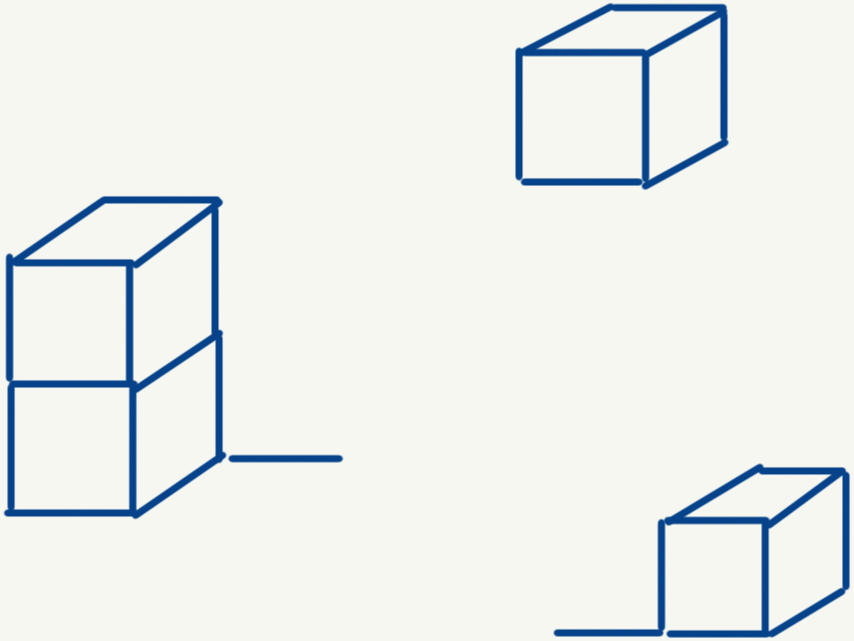
Procedure : While X is not an edge,

Apply ① if possible, then iterate on components.

Else apply ② if possible, then iterate.

Else apply ③ if possible, then iterate.

Else apply ④, then iterate.



Corrected proof (Reduction)

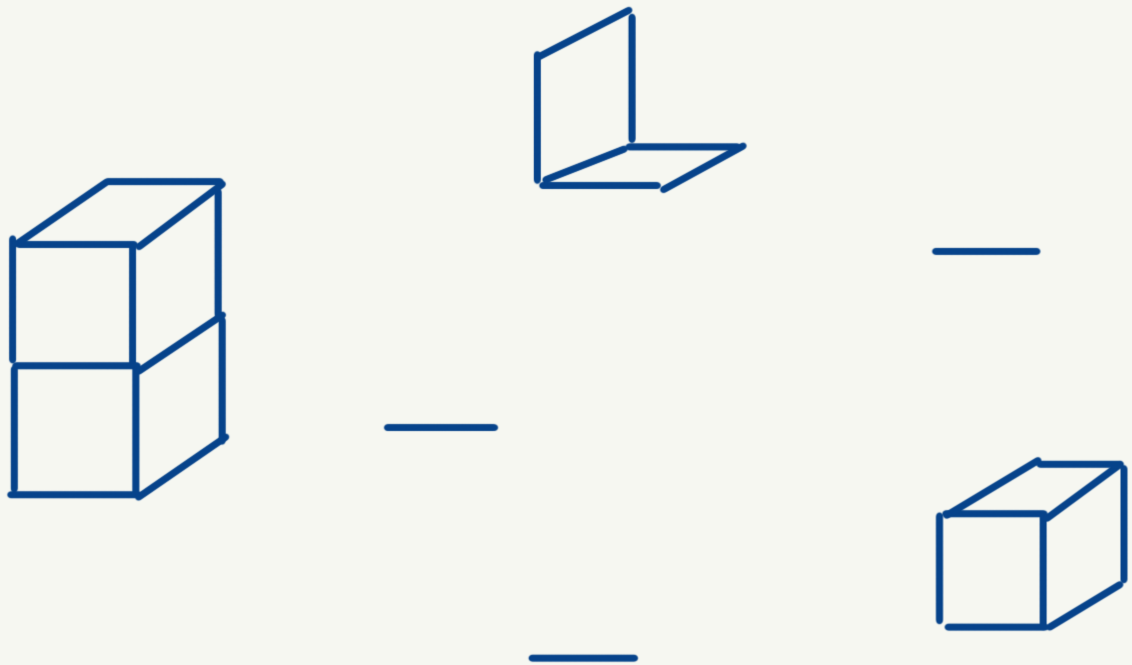
Procedure: While X is not an edge,

Apply 1 if possible, then iterate on components.

Else apply 2 if possible, then iterate.

Else apply 3 if possible, then iterate.

Else apply 4, then iterate.



Corrected proof (Reduction)

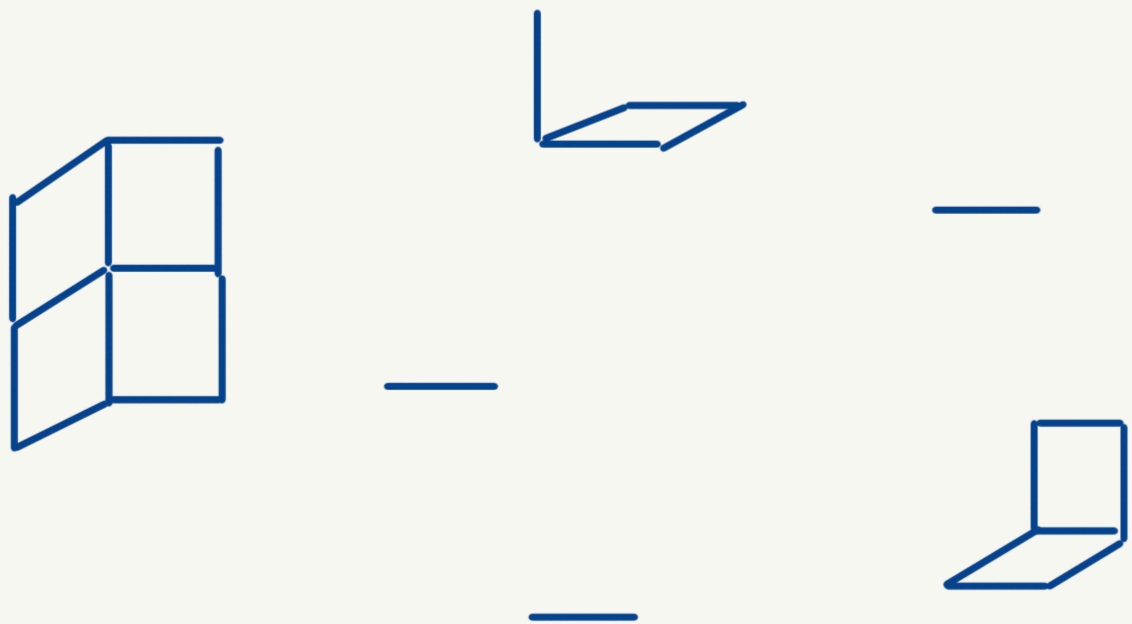
Procedure : While X is not an edge,

Apply ① if possible, then iterate on components.

Else apply ② if possible, then iterate.

Else apply ③ if possible, then iterate.

Else apply ④, then iterate.



Corrected proof (Reduction)

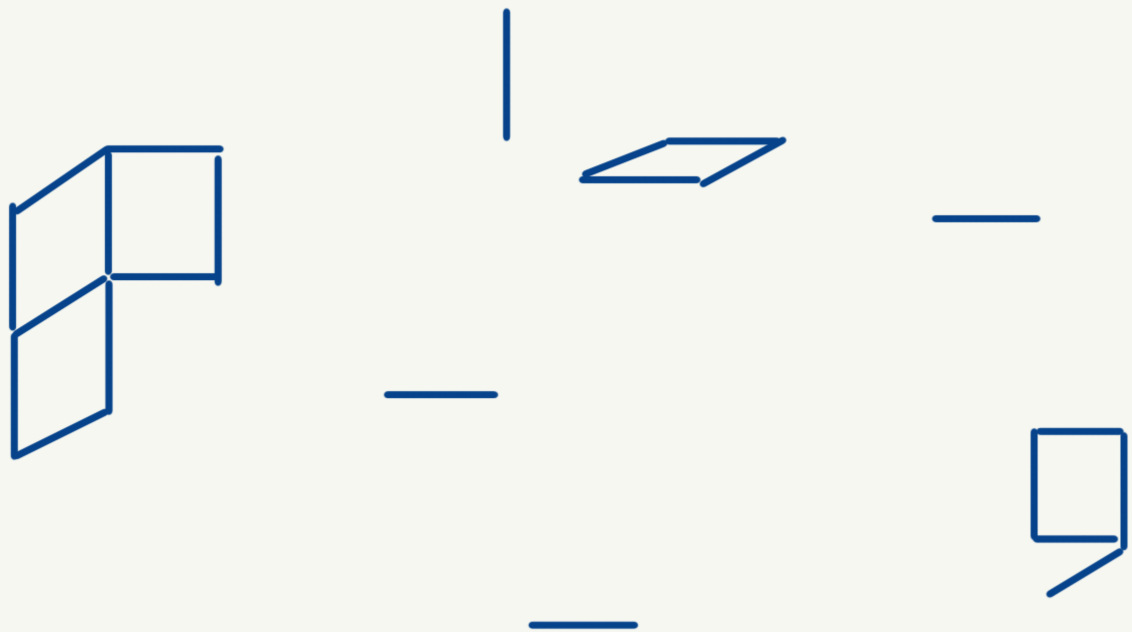
Procedure : While X is not an edge,

Apply ① if possible, then iterate on components.

Else apply ② if possible, then iterate.

Else apply ③ if possible, then iterate.

Else apply ④, then iterate.



Corrected proof (Reduction)

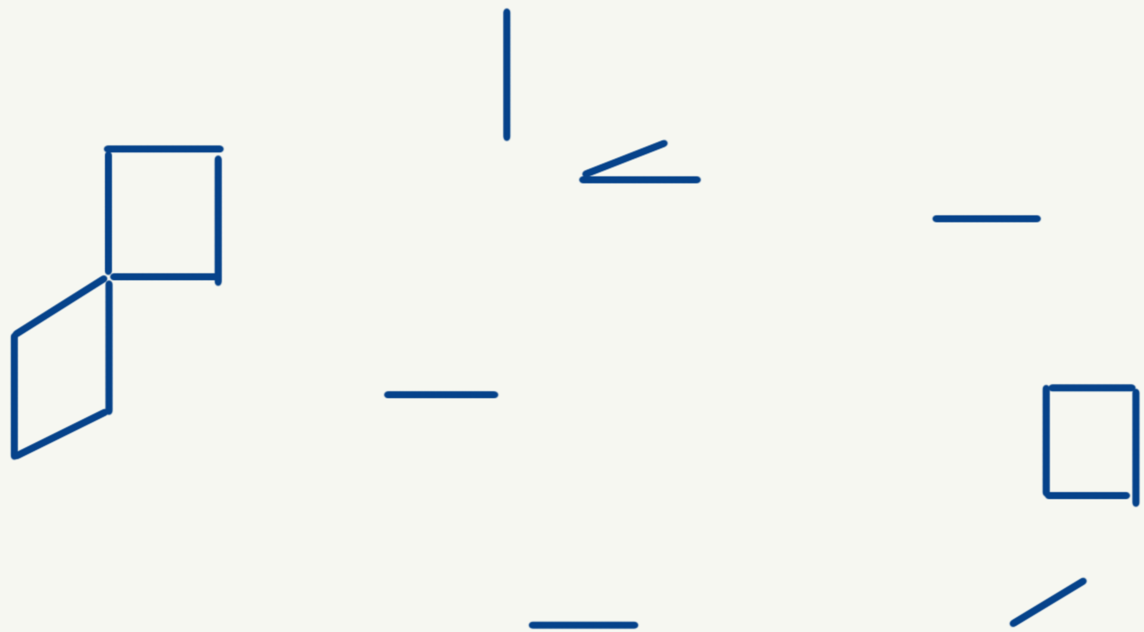
Procedure : While X is not an edge,

Apply ① if possible, then iterate on components.

Else apply ② if possible, then iterate.

Else apply ③ if possible, then iterate.

Else apply ④, then iterate.



Corrected proof (Reduction)

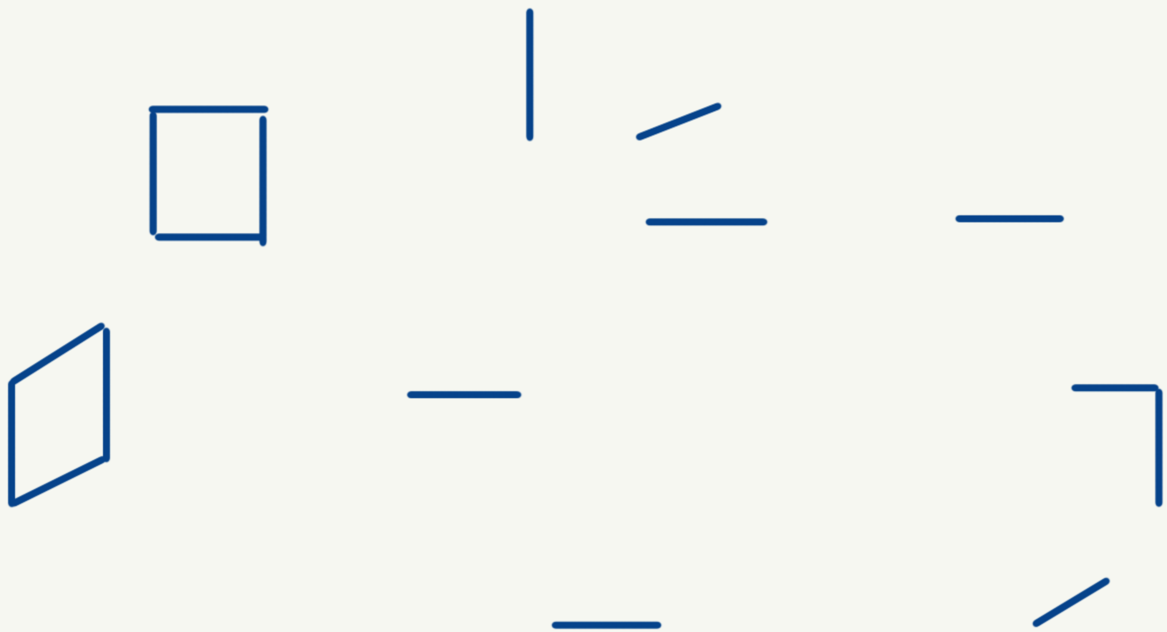
Procedure : While X is not an edge,

Apply 1 if possible, then iterate on components.

Else apply 2 if possible, then iterate.

Else apply 3 if possible, then iterate.

Else apply 4, then iterate.



Corrected proof (Reduction)

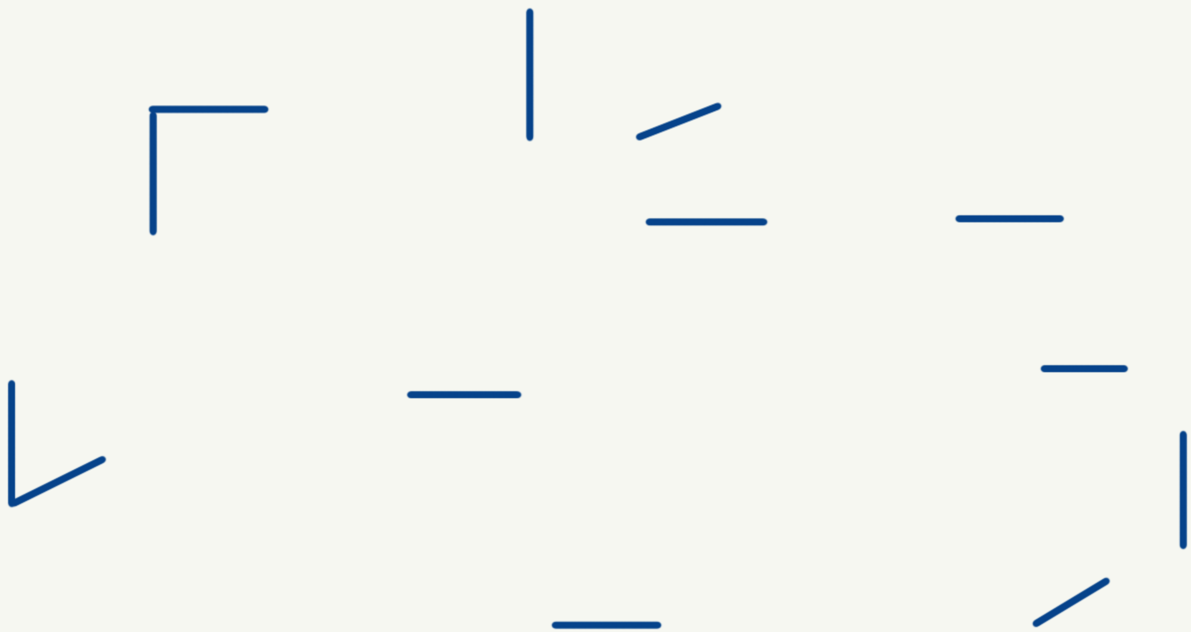
Procedure : While X is not an edge,

Apply 1 if possible, then iterate on components.

Else apply 2 if possible, then iterate.

Else apply 3 if possible, then iterate.

Else apply 4, then iterate.



Corrected proof (Reduction)

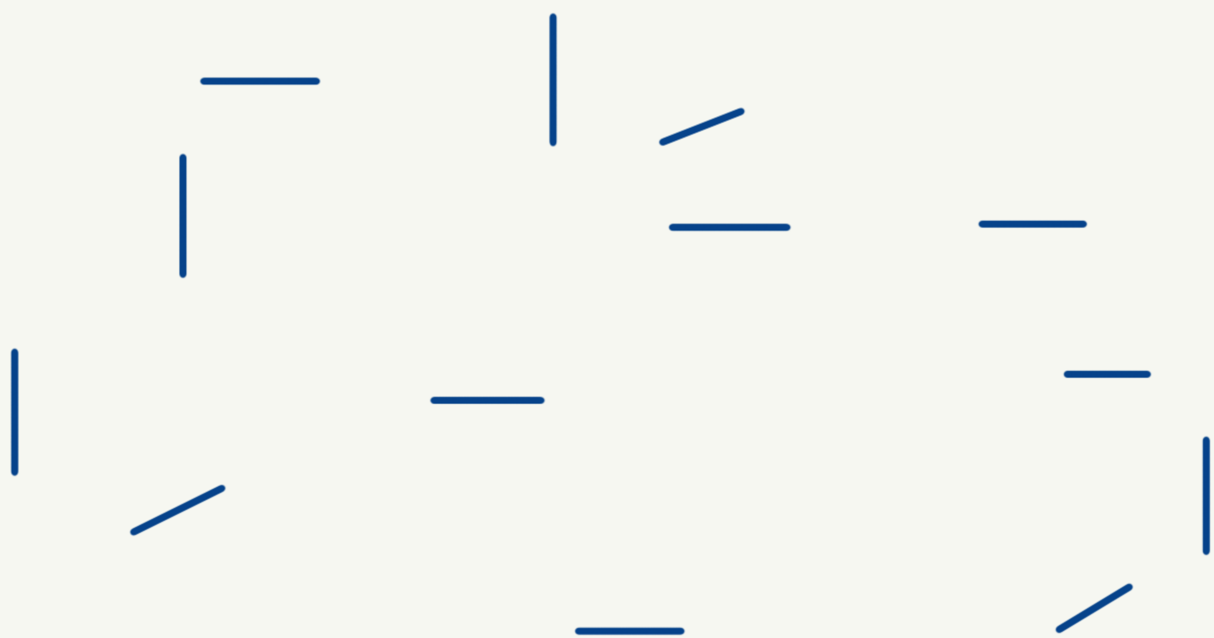
Procedure : While X is not an edge,

Apply 1 if possible, then iterate on components.

Else apply 2 if possible, then iterate.

Else apply 3 if possible, then iterate.

Else apply 4, then iterate.



Variations

simply connected
links are flag

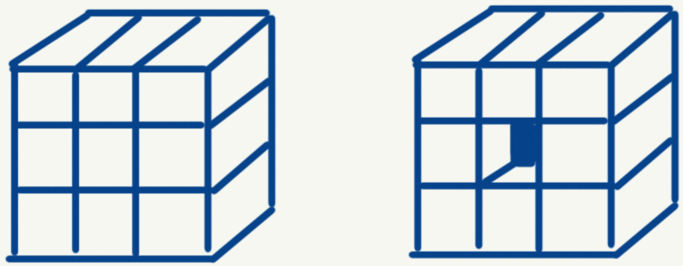
Our theorem is for finite, CAT(0) cube complexes embeddable in $\mathbb{R}^3$. What if we...

Variations

simply connected / links are flag

Our theorem is for finite, CAT(0) cube complexes embeddable in $\mathbb{R}^3$. What if we...

Remove simply connected

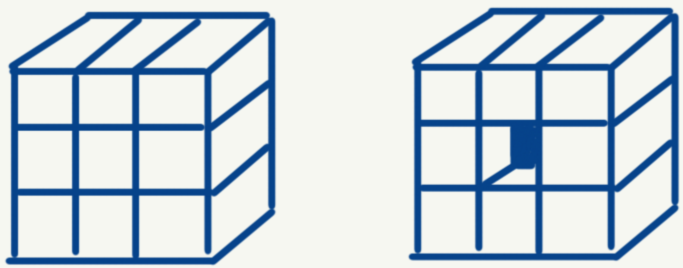


Variations

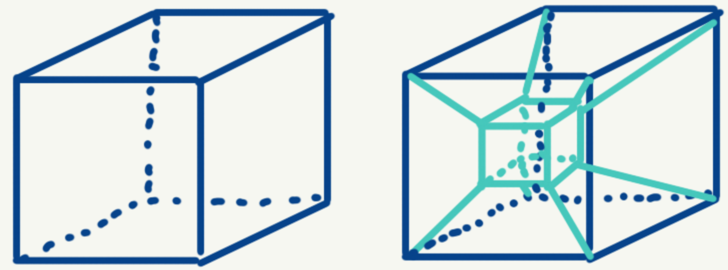
simply connected — links are flag

Our theorem is for finite, CAT(0) cube complexes embeddable in $\mathbb{R}^3$. What if we...

Remove simply connected



Remove flag

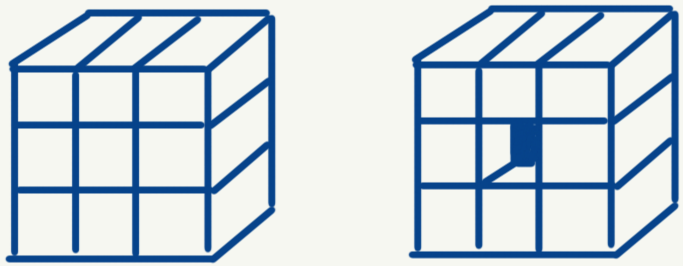


Variations

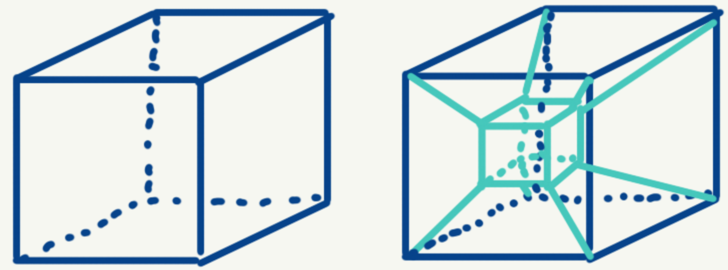
simply connected — links are flag

Our theorem is for finite, CAT(0) cube complexes embeddable in $\mathbb{R}^3$. What if we...

Remove simply connected



Remove flag



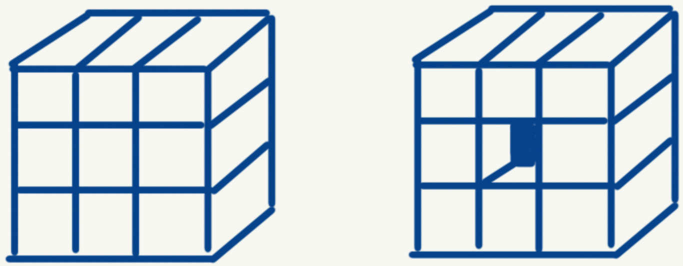
Conjecture: we should be able to reconstruct any finite CAT(0) cube complex that has an embedding in $\mathbb{R}^k$ from its matrix of boundary distances

Variations

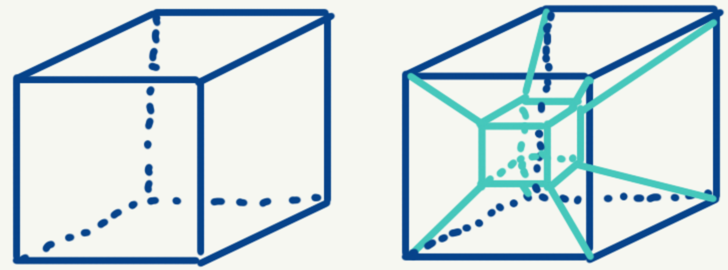
simply connected   links are flag

Our theorem is for finite, CAT(0) cube complexes embeddable in $\mathbb{R}^3$. What if we...

Remove simply connected



Remove flag



Conjecture we should be able to reconstruct any finite CAT(0) cube complex that has an embedding in $\mathbb{R}^k$ from its matrix of boundary distances

Question Under what conditions can we reconstruct a k-dimensional simplicial complex from its boundary distances?